

## CS 383

### HW 4

Due in class Friday, October 11 though it would help you to do the first four problems before taking Exam 1

This one should be typed.

1. Remember quotient languages from HW 3: If  $L$  is a regular language over  $\Sigma$  and  $a \in \Sigma$  then  $L/a$  is the set of strings  $w$  such that  $wa$  is in  $L$ . Either prove or disprove the following identities:
  - a.  $(L/a)a = L$
  - b.  $(La)/a = L$
2. Suppose  $L$  is a regular language. Show that  $\text{min}(L)$  is also regular, where  $\text{min}(L) = \{w \mid w \text{ is in } L \text{ but no proper prefix of } w \text{ is in } L\}$
3. Suppose  $L$  is regular. Show that  $\text{prefix}(L)$  is also regular, where  $\text{prefix}(L) = \{w \mid wx \text{ is in } L \text{ for some } x \text{ (including } x=\varepsilon)\}$ .  $\text{prefix}(L)$  is the set of all prefixes of all strings in  $L$ . These don't need to be proper prefixes, so  $L$  is a subset of  $\text{prefix}(L)$
4. For any language  $L$  let  $\text{powers}(L) = \{x^n \mid n \geq 0 \text{ and } x \in L\}$ . Find an example where  $L$  is regular but  $\text{powers}(L)$  is not regular.
5. Design a context-free grammar for  $\{0^n 1^n \mid n \geq 1\}$
6. Design a context-free grammar for  $\{a^i b^j c^k \mid i \neq j\}$
7. Here is a context-free grammar:

$$S \Rightarrow aS \mid Sb \mid a \mid b$$

Prove by induction on the string length that no string in the language represented by this grammar has  $ba$  as a substring.